FIND $7! = 7\cdot6\cdot\underbrace{5\cdot4\cdot3\cdot2\cdot1}_{5!} = 7\cdot6\cdot120$

$= 42\cdot120$

$= 5040$

NOTE: $n! = n\underbrace{(n-1)(n-2)\cdots1}_{(n-1)!} = n\cdot(n-1)!$

! HAS HIGHER PRECEDENCE THAN $*$ $\div$ $+$ $-$

$2n!$ VERSUS $(2n)!$

$= 2(n!)$ $\qquad = (2n)(2n-1)(2n-2)\cdots1$

$= 2\cdot n(n-1)(n-2)\cdots1$

IS $(A+B)! = A! + B!$? TRY $A=1$, $B=2$

$\boxed{\text{NO}}$

$(A+B)! = (1+2)! = 3! = 3\cdot2\cdot1 = 6$

$A! + B! = 1! + 2! = 1 + 2\cdot1 = 1+2 = 3$

IS $(AB)! = A!B!$? TRY $A=2$, $B=3$

$\boxed{\text{NO}}$

$(AB)! = (2\cdot3)! = 6! = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 720$

$A!B! = 2!3! = (2\cdot1)(3\cdot2\cdot1) = 2\cdot6 = 12$

$$\frac{11!}{8!} = \frac{11\cdot 10\cdot 9\cdot \cancel{8}\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}}{\cancel{8}\cdot \cancel{7}\cdot \cancel{6}\cdot \cancel{5}\cdot \cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot \cancel{1}} = 11\cdot 10\cdot 9 = 990$$

OR $\frac{11\cdot 10\cdot 9\cdot \cancel{8!}}{\cancel{8!}}$

$$\frac{n!}{(n+2)!} = \frac{\cancel{n!}}{(n+2)(n+1)\,\cancel{n!}}$$

$-1 \quad -1$

$$= \frac{1}{(n+2)(n+1)}$$

$$\frac{14!}{4!\,10!} = \frac{\overset{7}{\cancel{14}}\cdot 13\cdot \cancel{12}\cdot 11\cdot \cancel{10!}}{\cancel{4}\cdot \cancel{3}\cdot \cancel{2}\cdot 1\cdot \cancel{10!}} = 7\cdot 13\cdot 11 = 1001$$

COMMON SEQUENCES YOU SHOULD RECOGNIZE

1, 2, 4, 8, 16, 32 ← POWERS OF 2

1, 3, 9, 27, 81, 243 ← POWERS OF 3

2, 4, 6, 8, 10, 12, 14 ← MULTIPLES OF 2

3, 6, 9, 12, 15, 18, 21   3

⋮

12

1, 2, 6, 24, 120, 720, 5040

↑

FACTORIALS

1, 4, 9, 16, 25, 36, 49 ← PERFECT SQUARES

1, 8, 27, 64, 125, 216, 343   CUBES

SERIES

A SERIES IS THE SUM OF THE TERMS OF A SEQUENCE

$a = 1, 4, 9, 16, 25, 36$ ← SEQUENCE

40

$S_6 = 1 + 4 + 9 + 16 + 25 + 36$ ← SERIES

10

$= 10 + 40 + 41$ ← NOTE: $S_4 = 1 + 4 + 9 + 16$

$= 30$

$= 91$

GIVEN SEQUENCE $a_1, a_2, a_3, a_4, \ldots$

S_n = SUM OF 1ST n TERMS OF SEQUENCE

$= a_1 + a_2 + a_3 + \cdots + a_n$

CONSIDER $a_n = (n-1)(n-2)(n-3)(n-4) + n$

$a_1 = 1$

$a_2 = 2$

$a_3 = 3$

$a_4 = 4$

$a_5 = 4 \cdot 3 \cdot 2 \cdot 1 + 5 = 29$

SIGMA NOTATION

eg.

$$\sum_{n=3}^{7}(2n^2-n) = (2\cdot 3^2-3)+(2\cdot 4^2-4)+(2\cdot 5^2-5)+(2\cdot 6^2-6)+(2\cdot 7^2-7)$$

$$= 15+28+45+66+91$$

60

94

$$= 60+94+91$$

185

$$= 245$$

UPPER LIMIT OF SUMMATION

$$\sum_{n=a}^{b} f(n) = f(a)+f(a+1)+f(a+2)+\dots+f(b)$$

$[a, b \in \mathbb{Z}$ WHERE $a \leq b]$

DUMMY INDEX OF SUMMATION

LOWER LIMIT OF SUMMATION

$$\sum_{n=3}^{6} n^2 = 3^2+4^2+5^2+6^2 = \sum_{k=3}^{6} k^2 = 3^2+4^2+5^2+6^2$$

$$= \sum_{n=2}^{5}(n+1)^2 = 3^2+4^2+5^2+6^2$$

WRITE IN SIGMA NOTATION

(a) $16-25+36-49+64-81$

$16, -25, 36, -49, 64, -81$

$(-1)^4 4^2, (-1)^5 5^2, (-1)^6 6^2, (-1)^7 7^2, (-1)^8 8^2, (-1)^9 9^2$

$n=4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$

OR

$n=1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$$\sum_{n=4}^{9} (-1)^n n^2$$

OR $$\sum_{n=1}^{6} (-1)^{n+3} (n+3)^2$$

(b) $\frac{3}{2} - \frac{4}{4} + \frac{5}{8} - \frac{6}{16} + \frac{7}{32} - \frac{8}{64}$

$\frac{3}{2^1} \quad -\frac{4}{2^2} \quad \frac{5}{2^3} \quad -\frac{6}{2^4} \quad \frac{7}{2^5} \quad -\frac{8}{2^6}$

$n= \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6$

$$\sum_{n=1}^{6} (-1)^{n+1} \frac{n+2}{2^n}$$